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Dispersion of fine settling particles from an elevated source in an oscillatory turbulent flow

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Abstract

The present paper examines the stream-wise dispersion of suspended fine particles with settling velocities in an oscillatory turbulent shear flow with or without a non-zero mean over a rough-bed surface when the particles are being released from an elevated continuous source. A finite-difference implicit method is employed to solve the unsteady turbulent convective-diffusion equation. A combined scheme of central and four-point upwind differences is used to solve the steady state equation and the Alternating Direction Implicit (ADI) method is adopted for unsteady equation. It is shown how the mixing of settling particles is influenced by the tidal oscillatory current and the corresponding eddy diffusivity when the initial distribution of concentration regarded as a line-source. The vertical concentration profiles of suspended fine particles with settling velocities are presented for different downstream stations for various values of settling velocity and the frequency of the oscillation in tidal flow. For two-dimensional unsteady dispersion equation, the behaviour of iso-concentration lines for different values of settling velocity, frequency of the oscillation, dispersion time and releasing height is studied in terms of the relative importance of convection and eddy diffusion.

Keywords: Dispersion; Turbulent flow; Tidal flow; Line-source; ADI-method; Eddy diffusivity; Settling velocity

1. Introduction

The longitudinal dispersion of passive settling particles released from an elevated source in an oscillatory turbulent flow is worth studying from a practical point of view because of its numerous application in environmental problems. The present study gives an insight into the dispersion phenomena of passive suspended particles with settling velocities in turbulent flow and it has a great importance in industrial and technological fields.

The longitudinal dispersion of soluble matter in a viscous fluid flowing through a circular pipe under turbulent conditions was first studied by Taylor [1]. He observed that the longitudinal dispersion was dominated by the combined action of shearing current in the stream-wise direction and mixing in the cross-sectional plane. Elder [2] later applied Taylor's model to describe longitudinal dispersion in turbulent open-channel flow. Mazumder and Bandyopadhyay [3]

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presented a numerical solution to the convection–diffusion equation for solute dispersion in an open-channel flow with a modified 'log-wake-law' and corresponding eddy diffusivity due to Nezu and Nakagawa [4]. Their method provides a more general numerical study of time-dependent problem than the theoretical study of Sullivan and Yip [5] based on a small time asymptotic solution.

Aris [6] used his method of moments to study the longitudinal dispersion coefficient of solute in an oscillatory flow of a viscous incompressible fluid within an infinite tube under a periodic pressure gradient. Some important characteristics of the dispersion phenomenon in time-dependent flows within a conduit may be found in [7–15]. By considering an oscillatory two-dimensional channel flow with a linear velocity profile, Holley et al. [11] examined the periodicity of dispersion coefficient. Yasuda [13] studied the dispersion structure due to the oscillatory boundary layer and showed that the layer played a significant role in the dispersion of the pollutant in tidal flows. Allen [16] discussed in detail the dispersion of passive tracers in steady and oscillatory turbulent shear flows using a random walk technique to simulate numerically the space and time variation of material dispersing from a contaminant source.

Many researchers investigated theoretically as well as experimentally the longitudinal dispersion process of settling particles in a steady, uniform, two-dimensional shear flow [17–22] but the dispersion of suspended particles in an oscillatory turbulent shear flow has received scant attention despite the fact that such studies have a great practical importance. Wilson and Okubo [23] examined the dispersion of settling particles in an oscillatory current with a linear profile. Yasuda [24] presented the longitudinal dispersion of suspended fine particles with settling velocities for all time period in a tidal oscillatory current with a Stokes layer and showed that when the settling velocity exceeds a certain value, the dispersion coefficient of particles decreases with increase of settling velocity. Mei and Chian [25] examined theoretically the long-time dispersion of settling particles in a wave boundary layer considering the flow to be three-dimensional and calculated explicitly the correlation between velocity oscillations and concentration using constant eddy diffusivity.

The main objective of the present paper is to study the stream-wise dispersion of passive settling particles released from an elevated continuous line-source in an oscillatory turbulent flow. The results show how the injected particles disperses with the flow, how the spreading of settling particles are influenced by the combined action of oscillatory shear flow, settling velocity and the corresponding eddy diffusivity over the rough-surface for all times. A combined scheme of central differencing for diffusion terms and four-point upwind scheme for convection terms is used to solve the two-dimensional convection–diffusion equation for steady state and the Alternating Direction Implicit (ADI) scheme is employed for unsteady state. The vertical concentration profiles of suspended particles are presented for steady dispersion at different downstream stations for various values of settling velocity and frequency parameter for oscillatory flows with or without a non-zero mean. Results of two-dimensional unsteady dispersion are discussed in the form of iso-concentration contours in the vertical plane for different values of dispersion time, settling velocity, frequency parameter and releasing height. The interest in the analysis of dispersion of settling particles in an oscillatory turbulent flow has been motivated to understand the mass transport mechanism in coastal environment.

2. Mathematical formulations of the problem

In order to predict the dispersion of settling particles accurately in an estuary using any type of models, it is important to incorporate different factors such as: flow oscillation due to tidal current, changes of bottom topography of the estuary, salinity and settling of suspended particles and the effects of aspect ratio in a three-dimensional model. However, in this paper, we have restricted our attention to the combined effect oscillatory flow due to tidal action and the suspended fine particles with settling velocity, since this is often a complex process of large-scale dispersion of settling particles in estuaries.

Consider an unsteady, fully developed, unidirectional homogeneous turbulent flow in an estuary of uniform flow depth D. A Cartesian coordinate system is employed with x^*-axis along the flow and z^*-axis normal to the bed. The process is considered to be independent of the lateral position. Here, the flow is assumed to be uniform over the longitudinal direction and it varies only in the vertical z^*-direction and time t^*. When the passive settling particles are released into the above flow from an elevated continuous line-source, the concentration $C(x, z, t)$ of suspended particles satisfies the non-dimensional convection–diffusion equation of the form

$$\frac{\partial C}{\partial t} + u(z,t)\frac{\partial C}{\partial x} - \omega_s \frac{\partial C}{\partial z} = k_x(z)\frac{\partial^2 C}{\partial x^2} + \frac{\partial}{\partial z}\left(k_z(z)\frac{\partial C}{\partial z}\right) \tag{1}$$

with the dimensionless variables

$$x=\frac{x^*}{D},\quad z=\frac{z^*}{D},\quad t=\frac{t^*u_*}{D},\quad u=\frac{u^*}{u_*},\quad \omega_s=\frac{\omega_s^*}{u_*},\quad k_{ij}(z)=\frac{k_{ij}^*}{u_*D}.$$

Here u_* is the friction velocity (taken as reference velocity), ω_s is the settling velocity, D is the depth of the carrier fluid bounded below at $z=z_0$ and above at $z=1$, where z_0 is the equivalent bed roughness height. $k_{ij}(z)$ is the non-dimensional form of turbulent diffusivity tensor which is only function of z, where $k_{11}=k_x$, $k_{22}=k_z$. If the coordinate axes are chosen in such a way that these axes coincide with the principal axes of the turbulent fluctuations, the cross-variance become zero in the field of stationary homogeneous turbulence. So, in this study the off-diagonal terms in eddy-diffusivity tensor k_{ij} have been excluded (Fischer et al. [26]).

The dimensionless boundary conditions of the problem are

$$C(\pm\infty,z,t)=0,\qquad \left[k_z(z)\frac{\partial C}{\partial z}+\omega_s C\right]_{z=z_0,1}=0. \tag{2}$$

In addition, the unity concentration is maintained at the steady line-source ($x=0$, $z=z_s$) for all times. Thus, the physical problem is to seek the long-time diffusion of settling particles from the localized line-source.

Several investigations have been made using the oscillatory flow with or without a non-zero mean through different conduits. Lodahl et al. [27] have studied experimentally the transition of turbulence for steady, periodic and combined steady and periodic flows with constant amplitude through a pipe. However, for the present case, in order to study the space-time variation of released particles from an elevated line-source in an oscillatory turbulent flow, a simple two-dimensional model of the Mersey estuary is considered where the velocity profile represents a first order approximation to the form of an oscillatory flow (Allen [16]) as:

$$u(z,t)=u_0(z)+u_1(z)\sin\Omega t. \tag{3}$$

The first and second terms of Eq. (3) represent respectively the steady and unsteady components of the flow. The steady component of $u(z,t)$ represents the fresh water discharge etc., and the unsteady component represents the tidal oscillation with a frequency Ω corresponding to that of the semi-diurnal tide and the tidal amplitude $u_1(z)$. Here, dimensionless frequency $\Omega=(D/u_*).(1/(1/\Omega^*))$ is a measure of the ratio of the characteristic time (D/u_*) due to transverse diffusion to the period of oscillation $(1/\Omega^*)$. If the model is extended to three-dimension, the $\sin\Omega t$ term may be modified as $\sin(\Omega t+\phi)$, where ϕ, is a function of z, is the phase lag term. The steady non-dimensional form of velocity distribution $u_0(z)$ over the rough bed surface is considered as

$$u_0(z)=\frac{1}{\kappa}\ln\left(\frac{z}{z_0}\right)+W(z), \tag{4}$$

where κ is the von-Kármán constant, z_0 is the equivalent bed roughness and the wake-function $W(z)$ is taken as

$$W(z)=\frac{2\Pi}{\kappa}\sin^2\left(\frac{\pi}{2}z\right) \tag{5}$$

where Π is the wake-strength parameter. The term 'wake-function' is added to the log-law in the outer region to incorporate the effect of wakes generated below the free surface and it helps to get the closer agreement with the standard 'log-law' (Coles [28]). It may be mentioned here that the 'log-wake law' can be used only in fully developed homogeneous turbulent flow. The tidal amplitude $u_1(z)$ of the unsteady part of Eq. (3) is a simple parabolic distribution suggested by Bowden and Fairbain [29] as

$$u_1(z)=1.15U_1(0.63+0.37z^2) \tag{6}$$

where U_1 is the depth-averaged tidal amplitude taken from Allen [16]. The eddy-diffusivity is taken here as space variable similar to that of Nezu and Rodi [30] is given by

$$k(z)=\kappa(1-z)\left[\frac{1}{z}+\Pi\pi\sin(\pi z)\right]^{-1}. \tag{7}$$

The aim of the present study is to solve Eq. (1) numerically subject to the boundary conditions (2) and the prescribed input condition for the passive settling particles. First, the steady state of the convection–diffusion equation (1) is solved by taking steady flow $u_0(z)$ with zero settling velocity ($\omega_s = 0$) and results are compared with the experimental data of Raupach and Legg [31] and also with the numerical results of Sullivan and Yip [32]. Finally, the two-dimensional unsteady convection–diffusion equation is solved numerically using the prescribed oscillatory turbulent velocity profile with or without a non-zero mean for variable eddy diffusivity.

3. Numerical procedure

In order to discuss the dispersion phenomena of the released particles into the oscillatory turbulent flow, Eq. (1) together with the boundary conditions (2) and prescribed input condition has been solved numerically using a finite difference technique. According to the formulation of the problem, it is not convenient to incorporate the boundary conditions at infinity. To avoid this difficulty along x-direction, a transformation is taken to map the unbounded region [physical plane (x, z)] to a bounded one [computational plane (ζ, η)]. The transformations used in this problem are of the form

$$x = \frac{1}{2a}\log\left(\frac{1+\zeta}{1-\zeta}\right), \quad \text{and} \quad z = \eta \tag{8}$$

for

$$-1 < \zeta < 1, \qquad \eta_0 \leqslant \eta \leqslant 1.$$

Here a is the stretching factor relating the physical domain to the computational domain. This form of transformation is needed to avoid the loss of accuracy through dicretization in the diffusion and convection terms. Using the transformation (8), Eq. (1) and the respective boundary conditions (2) in the computational plane become

$$\frac{\partial C}{\partial t} + u(\eta, t)a(1-\zeta^2)\frac{\partial C}{\partial \zeta} - \omega_s\frac{\partial C}{\partial \eta} = a^2 k_\zeta(\eta)(1-\zeta^2)\left[(1-\zeta^2)\frac{\partial^2 C}{\partial \zeta^2} - 2\zeta\frac{\partial C}{\partial \zeta}\right] + \frac{\partial}{\partial \eta}\left(k_\eta(\eta)\frac{\partial C}{\partial \eta}\right) \tag{9}$$

and

$$C(\pm 1, \eta, t) = 0, \qquad \left[k_\eta(\eta)\frac{\partial C}{\partial \eta} + \omega_s C\right]_{\eta=\eta_0, 1} = 0. \tag{10}$$

Since the flow is turbulent and fully-developed, the convection in the ζ-direction is much larger than the longitudinal diffusion of the settling particles. If the particles are injected at the line $\zeta = 0$, it cannot reach at the negative side of the line ($\zeta = 0$) because of the dominance of the convection effect. Hence the concentration of the particles for $-1 \leqslant \zeta \leqslant 0$ is assumed to be zero.

3.1. Steady-state concentration equation

The steady-state form of convection–diffusion equation (9) is written as

$$u(\eta, t)a(1-\zeta^2)\frac{\partial C}{\partial \zeta} - \omega_s\frac{\partial C}{\partial \eta} - a^2 k_\zeta(\eta)(1-\zeta^2)\left[(1-\zeta^2)\frac{\partial^2 C}{\partial \zeta^2} - 2\zeta\frac{\partial C}{\partial \zeta}\right] - \frac{\partial}{\partial \eta}\left(k_\eta(\eta)\frac{\partial C}{\partial \eta}\right) = 0 \tag{11}$$

with the boundary conditions are

$$C(1, \eta) = 0, \qquad \left[k_\eta(\eta)\frac{\partial C}{\partial \eta} + \omega_s C\right]_{\eta=\eta_0, 1} = 0. \tag{12}$$

Here the input condition at the line source ($\zeta = 0, \eta = z_s$) is assumed to be unity for all times. In order to solve Eq. (11) subject to the prescribed boundary and input conditions, a combined scheme of central differencing and four-point upwind scheme is employed. Central differencing is used for diffusion terms whereas for the convection term the four-point upwind scheme is adopted because of the oscillatory nature of three-point central difference representation and the dissipative behaviour of two-point upwind scheme. So, a four-point upwind scheme for the convective terms $u\frac{\partial C}{\partial \zeta}$ and $\omega_s\frac{\partial C}{\partial \eta}$ is used and produces less error (Fletcher [33]). The following discretization is used for $\frac{\partial C}{\partial \zeta}$ at (j, k) grid point: for $u > 0$

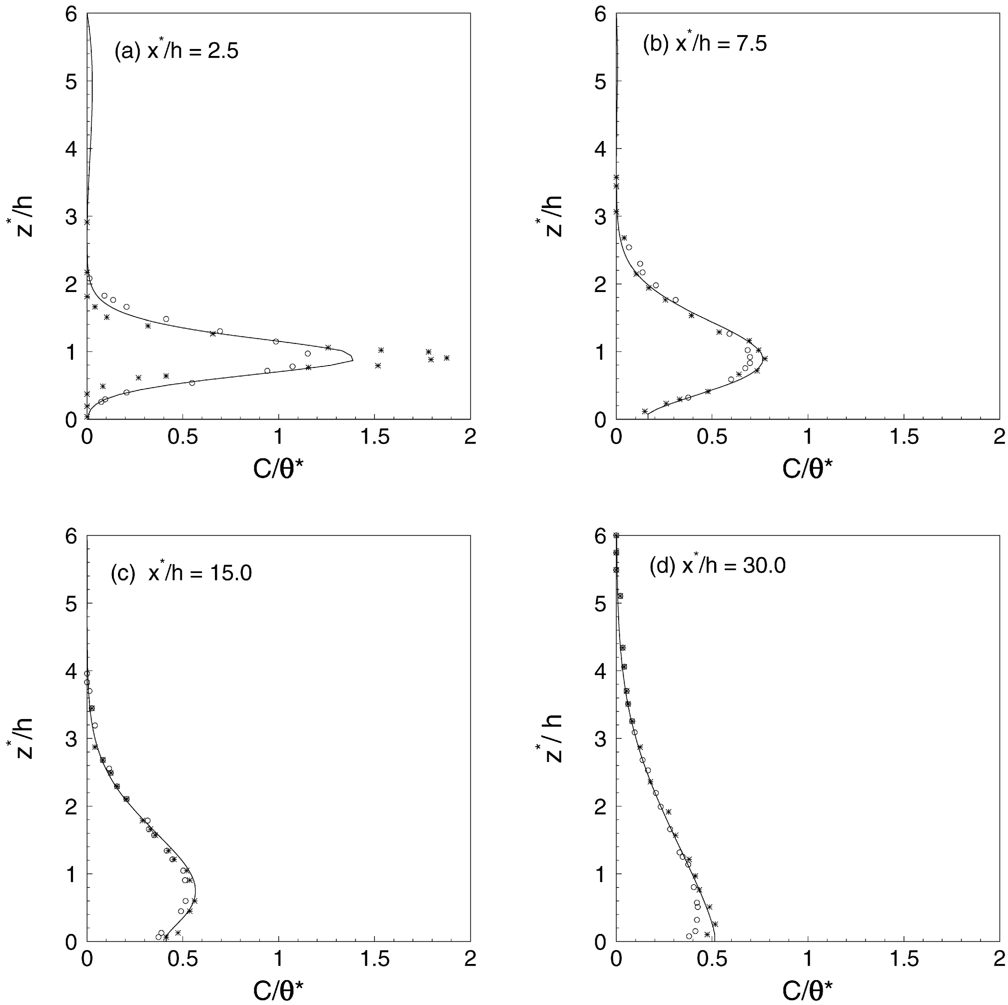

Fig. 1. Comparison of normalized steady concentration profiles at various downstream distances among the experiments of Raupach and Legg [26] ($* * *$), the solution-scheme of Sullivan and Yip [27] ($\circ \circ \circ$), and the present solution ($-$).

$$\left.\frac{\partial C}{\partial \zeta}\right|_{jk} = \frac{C(j+1,k) - C(j-1,k)}{2\Delta\zeta} + q_1\left[\frac{C(j-2,k) - 3C(j-1,k) + 3C(j,k) - C(j+1,k)}{3\Delta\zeta}\right] + \mathrm{O}(\Delta\zeta^2) \tag{13}$$

and for $u < 0$

$$\left.\frac{\partial C}{\partial \zeta}\right|_{jk} = \frac{C(j+1,k) - C(j-1,k)}{2\Delta\zeta} + q_1\left[\frac{C(j-1,k) - 3C(j,k) + 3C(j+1,k) - C(j+2,k)}{3\Delta\zeta}\right] + \mathrm{O}(\Delta\zeta^2) \tag{14}$$

and the discretized form of $\frac{\partial C}{\partial \eta}$ is represented as: for $\omega_s > 0$

$$\left.\frac{\partial C}{\partial \eta}\right|_{jk} = \frac{C(j,k+1) - C(j,k-1)}{2\Delta\eta} + q_2\left[\frac{C(j,k-2) - 3C(j,k-1) + 3C(j,k) - C(j,k+1)}{3\Delta\eta}\right] + \mathrm{O}(\Delta\eta^2) \tag{15}$$

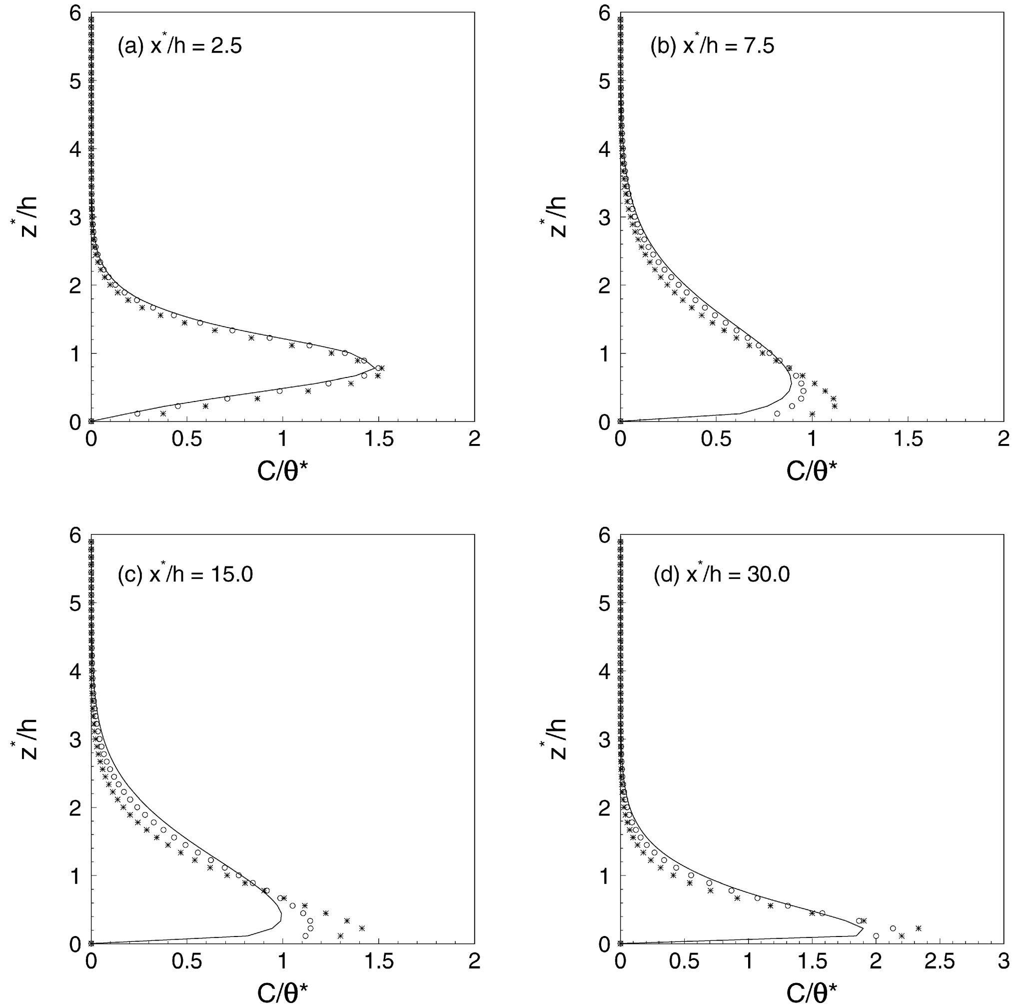

Fig. 2. Normalized steady concentration profiles for purely oscillatory flow with frequency $\Omega = 4$ at various downstream distances for $\omega_s = 0.1$ (—), $\omega_s = 0.2$ (○ ○ ○), and $\omega_s = 0.4$ (∗ ∗ ∗) at time $t = 2.0$.

and for $\omega_s < 0$,

$$\left.\frac{\partial C}{\partial \eta}\right|_{jk} = \frac{C(j,k+1) - C(j,k-1)}{2\Delta\eta} + q_2\left[\frac{C(j,k-1) - 3C(j,k) + 3C(j,k+1) - C(j,k+2)}{3\Delta\eta}\right] + \mathrm{O}(\Delta\eta^2). \tag{16}$$

The parameters q_1 and q_2 controls the size of the modification of three-point central finite difference formula and it is effective to reduce the dispersion error. On a finer grid, a suitable choice of q_1 and q_2 produces comparatively more accurate and non-oscillatory results than the other simple schemes. $C(j,k)$ indicates the value of the concentration of the settling particles at the mesh point (j,k): $\zeta = (j-1) \times \Delta\zeta$ and $\eta = \eta_0 + (k-1) \times \Delta\eta$, $\Delta\zeta$ and $\Delta\eta$ are the grid spacing along the x-axis and z-axis respectively. The mesh point, where the particles are injected, is described as $(1, k_p)$ where $j = 1, j = N+1$ correspond to $\zeta = 0, \zeta = 1$; $k = 1, k = M+1$ correspond to $\eta = \eta_0, \eta = 1$ and $k = k_p$ corresponds to $\eta = z_s$. N and M represents the maximum number of grid spacings respectively along ζ and η directions.

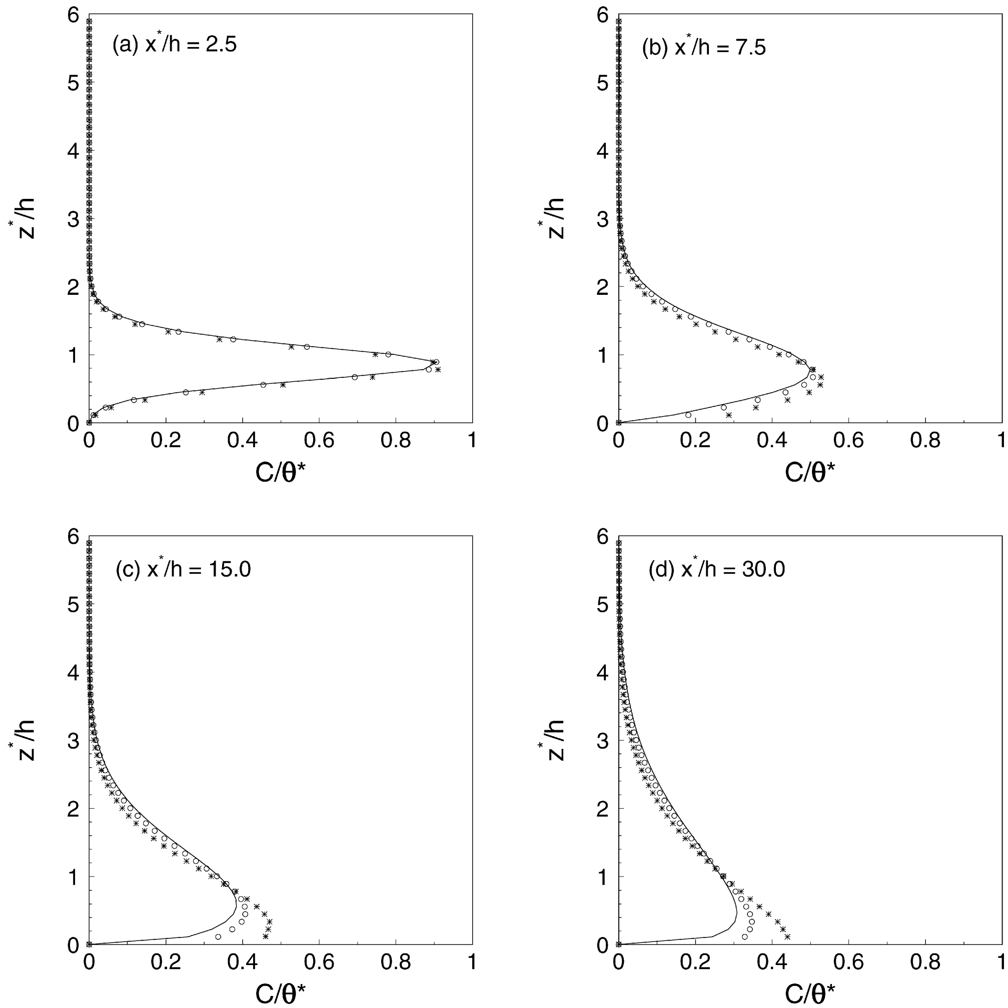

Fig. 3. Normalized steady concentration profiles for oscillatory flow of frequency $\Omega = 4$ with non-zero mean at various downstream distances for $\omega_s = 0.1$ (—), $\omega_s = 0.2$ (∘∘∘), and $\omega_s = 0.4$ (∗∗∗) at time $t = 2.0$.

The boundary conditions are then reduced to

$$\left.\begin{aligned} &C(j,k) = 0 \quad \text{for } 1 \leqslant j \leqslant N+1,\, 1 \leqslant k \leqslant M+1 \text{ except } k = k_p,\\ &C(j,0) = C(j,2) + \frac{2\omega_s \Delta\eta}{k_\eta(\eta)} C(j,1),\\ &C(j,M+2) = C(j,M) - \frac{2\omega_s \Delta\eta}{k_\eta(\eta)} C(j,M+1) \quad \text{for } 1 \leqslant j \leqslant N+1 \end{aligned}\right\} \tag{17}$$

and the prescribed input condition is $C(1,k_p) = 1$. The following conditions have been used for accurate and stable results:

$$\Delta\zeta = \left.\frac{k_\zeta(\eta)}{u(\eta,t)}\right|_{\min} \quad \forall j,k,$$

$$\Delta\eta = \left.\frac{k_\eta(\eta)}{u(\eta,t)}\right|_{\min} \quad \forall j,k,$$

and

$$q_1 = \frac{1}{2}\left[1 + \frac{a\zeta k_\zeta(\eta)}{u(\eta,t)}\right],$$

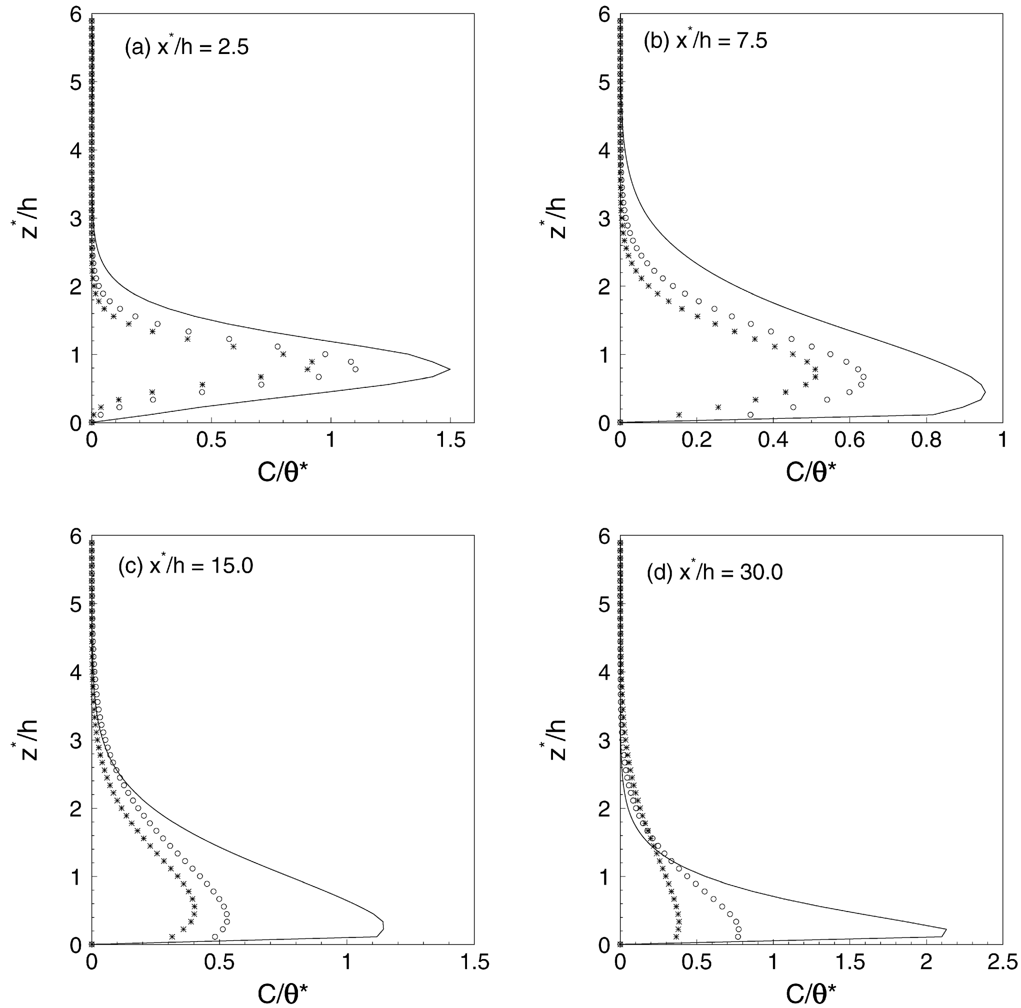

Fig. 4. Normalized steady concentration profiles of settling velocity $\omega_s = 0.2$ for purely oscillatory flow at various downstream distances for $\Omega = 4$ (—), $\Omega = 7$ (○ ○ ○), and $\Omega = 10$ (∗ ∗ ∗) at time $t = 2.0$.

$$q_2 = \frac{1}{2}\left[1 + \frac{a\eta k_\eta(\eta)}{u(\eta, t)}\right].$$

An inverse transformation has been applied to go back from the computational plane to the physical plane and to obtain the desired result after solving the discretized system of algebraic equations by means of the Successive Over Relaxation (SOR) method. The relaxation parameter has been found by numerical inspection.

3.2. Unsteady concentration equation

The unsteady two-dimensional form of convection–diffusion equation (9) is

$$\frac{\partial C}{\partial t} + u(\eta, t)a(1-\zeta^2)\frac{\partial C}{\partial \zeta} - \omega_s\frac{\partial C}{\partial \eta} - a^2k_\zeta(\eta)(1-\zeta^2)\left[(1-\zeta^2)\frac{\partial^2 C}{\partial \zeta^2} - 2\zeta\frac{\partial C}{\partial \zeta}\right] - \frac{\partial}{\partial \eta}\left(k_\eta(\eta)\frac{\partial C}{\partial \eta}\right) = 0 \quad (18)$$

with respective boundary (10) and prescribed input conditions. An Alternating Direction Implicit (ADI) method is adopted to solve Eq. (18). The ADI-scheme in two-dimensional case is unconditionally stable, second-order accurate and also economical to formulate. On solving Eq. (18), central differencing is used for diffusion terms and backward differencing for convective terms. This finite difference technique prevents the numerical oscillations to the solution, though the local spatial resolution is only of first order. In this scheme dispersive error may occur and it can be

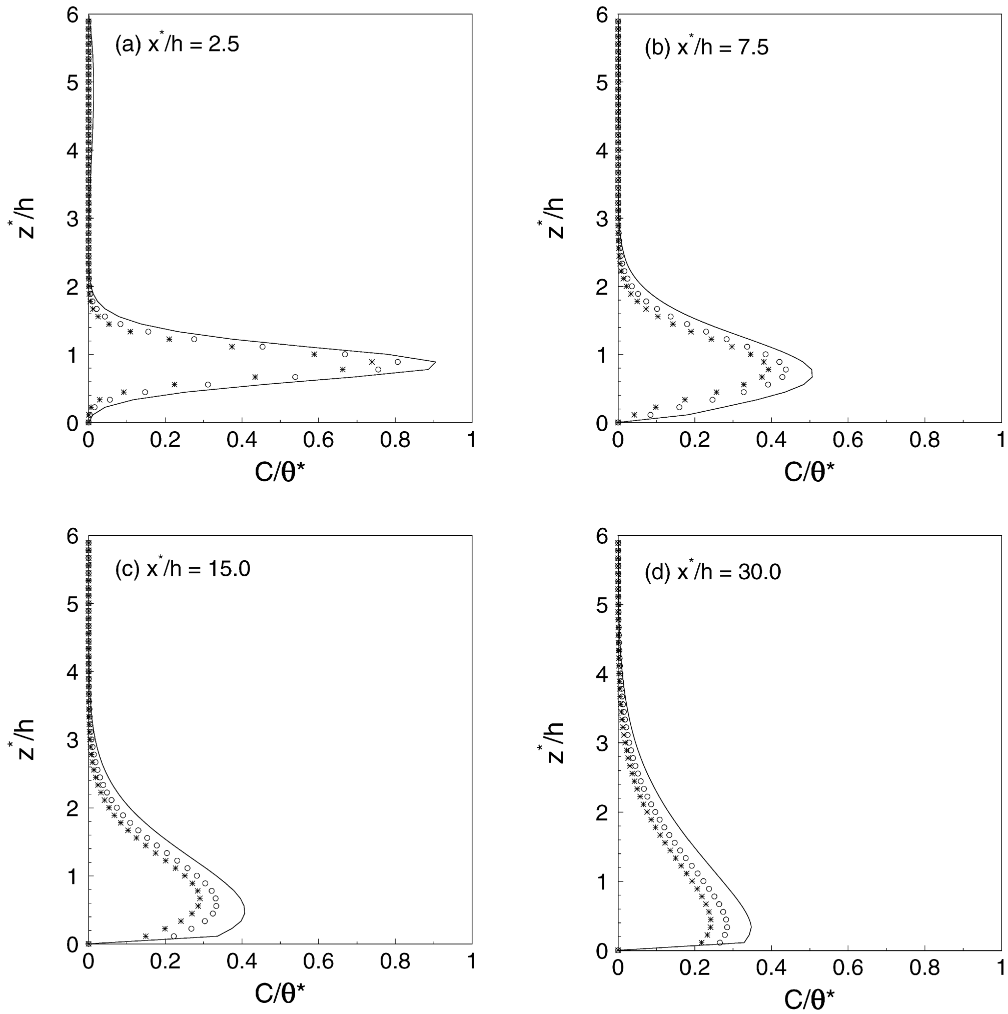

Fig. 5. Normalized steady concentration profiles of settling velocity $\omega_s = 0.2$ for oscillatory flow with non-zero mean at various downstream distances for $\Omega = 4$ (—), $\Omega = 7$ (○ ○ ○), and $\Omega = 10$ ($* * *$) at time $t = 2.0$.

overcome with mesh refinement which is not serious for the present two-dimensional unsteady advection–diffusion equation. We have adjusted the mesh refinement such that the solutions of steady concentration equation are in good agreement with the experimental data of Raupach and Legg [31] and numerical results of Sullivan and Yip [32]. In the present problem, we have considered $N = 40$ and $M = 40$ representing the maximum number of grid spacings respectively along ζ and η directions gives the satisfactory results and we have extended our work for long time dispersion in an unsteady flow. However, Karaa and Zhang [34] and You [35] proposed a higher order ADI-method for unsteady convection–diffusion equation, which may be taken as an alternative approach to studying the problem. Eq. (18) is written in two-half step with one spatial variable implicit in one-half step and other spatial variable implicit in the next-half step. Therefore, each half-step involves the direct solution of a tri-diagonal system of equations. During the first-half step the value of the concentration C is known at time level n but is unknown at the $(n + \frac{1}{2})$ time level, denoted by $*$. However, these unknown nodal values are associated with the ζ-direction only (i.e. for ζ-implicit and η-explicit). Eq. (18) can be discretized as

$$K_j C(j-1,k,*) + L_j C(j,k,*) + M_j C(j+1,k,*) = N_j \tag{19}$$

where

$$K_j = \frac{1}{2}\left[au(\eta,t)(1-\zeta^2)\frac{\Delta t}{\Delta\zeta} + a^2 k_\zeta(\eta)(1-\zeta^2)^2\frac{\Delta t}{(\Delta\zeta)^2} + a^2 k_\zeta(\eta)\zeta(1-\zeta^2)\frac{\Delta t}{\Delta\zeta}\right],$$

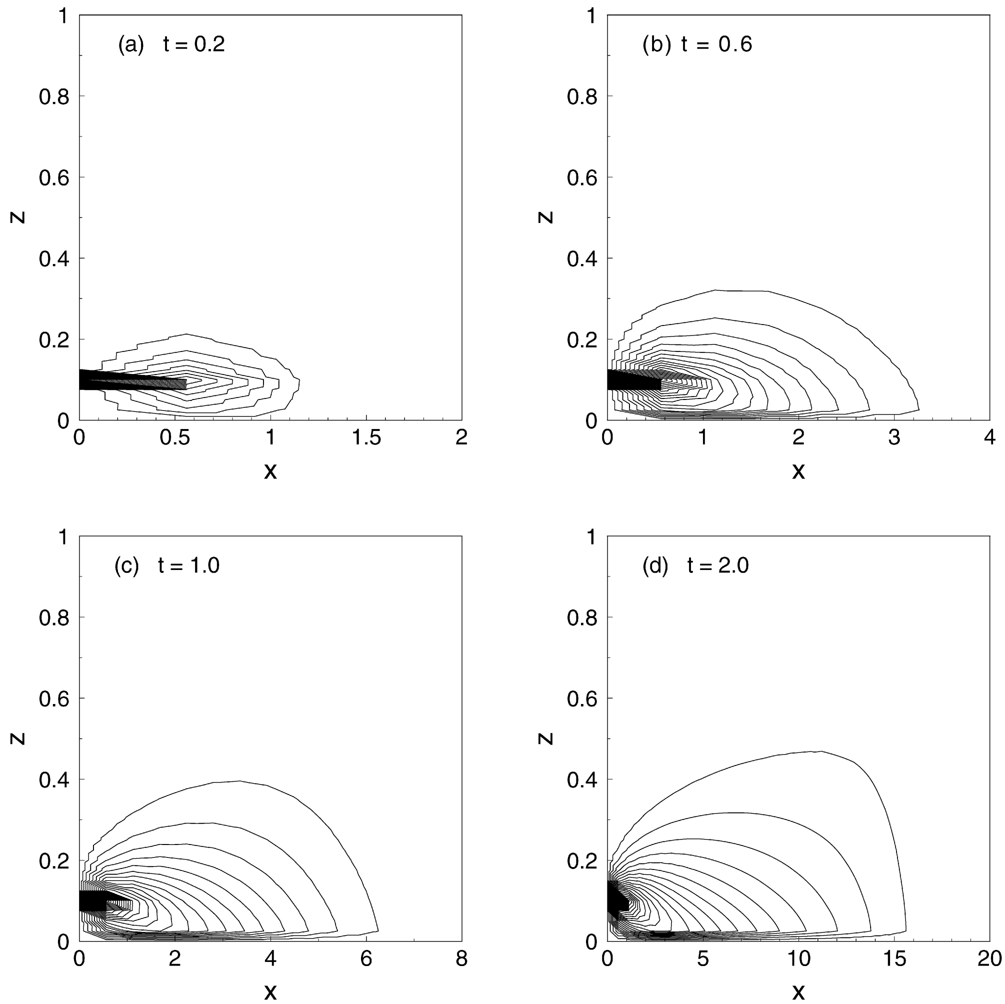

Fig. 6. Iso-concentration lines for unsteady dispersion in purely oscillatory flow with frequency $\Omega = 4$ when the particles of settling velocity $\omega_s = 0.2$ released at a height $z_s = 0.11$, for different non-dimensional times. The strength of the concentration of settling particles is 0.01 at the outermost contour.

$$L_j = -\left[1 + \frac{1}{2}au(\eta, t)(1-\zeta^2)\frac{\Delta t}{\Delta\zeta} + a^2 k_\zeta(\zeta)(1-\zeta^2)^2\frac{\Delta t}{(\Delta\zeta)^2}\right],$$

$$M_j = \frac{1}{2}a^2 k_\zeta(\eta)(1-\zeta^2)\left[(1-\zeta^2)\frac{\Delta t}{(\Delta\zeta)^2} - \zeta\frac{\Delta t}{\Delta\zeta}\right],$$

and

$$N_j = \left[\frac{k'_\eta(\eta)}{4}\frac{\Delta t}{\Delta\eta} - \frac{k_\eta(\eta)}{2}\frac{\Delta t}{(\Delta\eta)^2} + \frac{\omega_s}{2}\frac{\Delta t}{\Delta\eta}\right]C(j, k-1, n) + \left[-1 + k_\eta(\eta)\frac{\Delta t}{(\Delta\eta)^2} - \frac{\omega_s}{2}\frac{\Delta t}{\Delta\eta}\right]C(j, k, n)$$
$$- \left[\frac{k'_\eta(\eta)}{4}\frac{\Delta t}{\Delta\eta} + \frac{k_\eta(\eta)}{2}\frac{\Delta t}{(\Delta\eta)^2}\right]C(j, k+1, n).$$

Using Thomas algorithm (Anderson et al. [36]), Eq. (19), which is a tri-diagonal system of algebraic equations is solved for $C(j, k, *)$, $j = 2, \ldots, N$, for each row and $k = 1, \ldots, M+1$.

For next-half step with η-implicit and ζ-explicit, the following set of equations are solved for the unknown values of concentration $C(j, k, n+1)$ using the known intermediate values $C(j, k, *)$

$$K_k C(j, k-1, n+1) + L_k C(j, k, n+1) + M_k C(j, k+1, n+1) = N_k \tag{20}$$

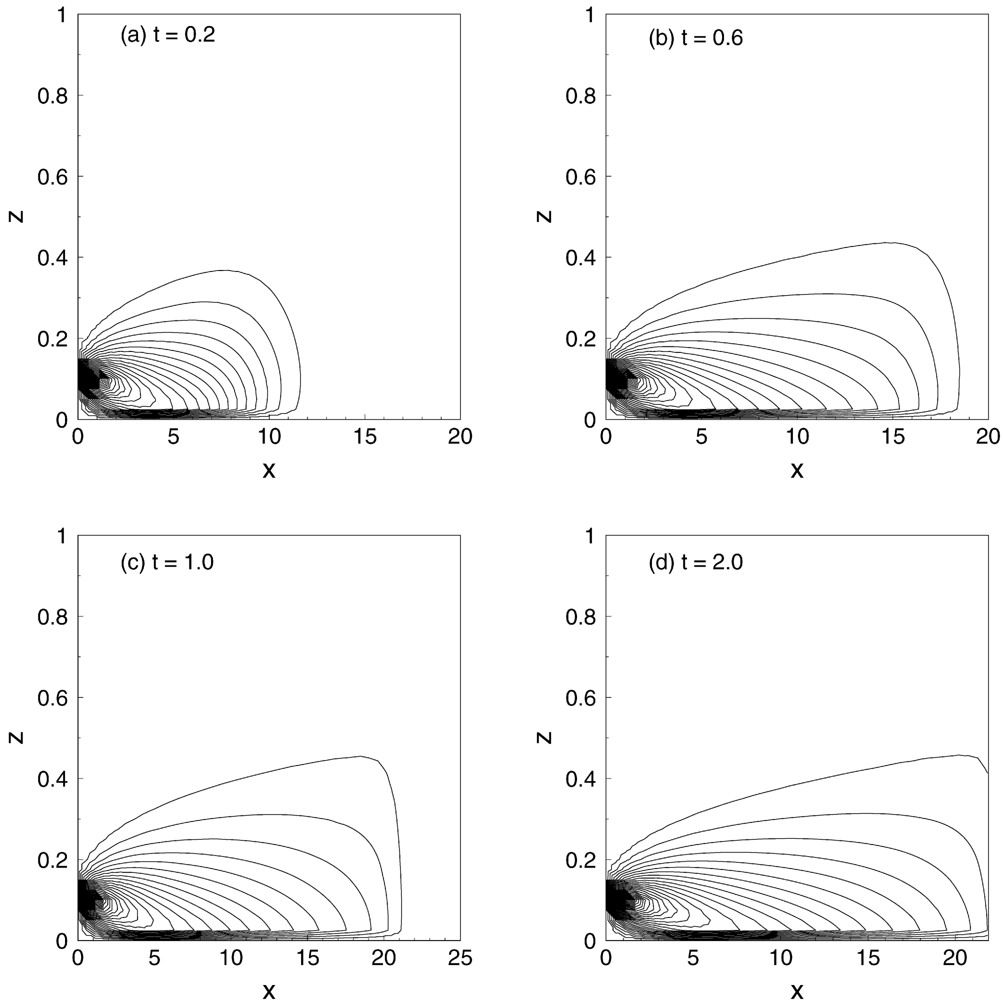

Fig. 7. Iso-concentration lines for unsteady dispersion for the case of oscillatory flow with non-zero mean. All other parameters are same as Fig. 6.

where

$$K_k = \left[\frac{k_\eta(\eta)}{2}\frac{\Delta t}{(\Delta\eta)^2} - \frac{k'_\eta(\eta)}{4}\frac{\Delta t}{\Delta\eta} - \frac{\omega_s}{2}\frac{\Delta t}{\Delta\eta}\right],$$
$$L_k = \left[-1 - k_\eta(\eta)\frac{\Delta t}{(\Delta\eta)^2} + \frac{\omega_s}{2}\frac{\Delta t}{\Delta\eta}\right],$$
$$M_k = \left[\frac{k'_\eta(\eta)}{4}\frac{\Delta t}{\Delta\eta} + \frac{k_\eta(\eta)}{2}\frac{\Delta t}{(\Delta\eta)^2}\right],$$

and

$$\begin{aligned}N_k = {} & -\frac{1}{2}\left[au(\eta,t)(1-\zeta^2)\frac{\Delta t}{\Delta\zeta} + a^2k_\zeta(\eta)(1-\zeta^2)^2\frac{\Delta t}{(\Delta\zeta)^2} + a^2k_\zeta(\eta)\zeta(1-\zeta^2)\frac{\Delta t}{\Delta\zeta}\right]C(j-1,k,*)\\ & + \left[-1 + \frac{1}{2}au(\eta,t)(1-\zeta^2)\frac{\Delta t}{\Delta\zeta} + a^2k_\zeta(\eta)(1-\zeta^2)^2\frac{\Delta t}{(\Delta\zeta)^2}\right]C(j,k,*)\\ & - \frac{1}{2}a^2k_\zeta(\eta)(1-\zeta^2)\left[(1-\zeta^2)\frac{\Delta t}{(\Delta\zeta)^2} - \zeta\frac{\Delta t}{\Delta\zeta}\right]C(j+1,k,*).\end{aligned}$$

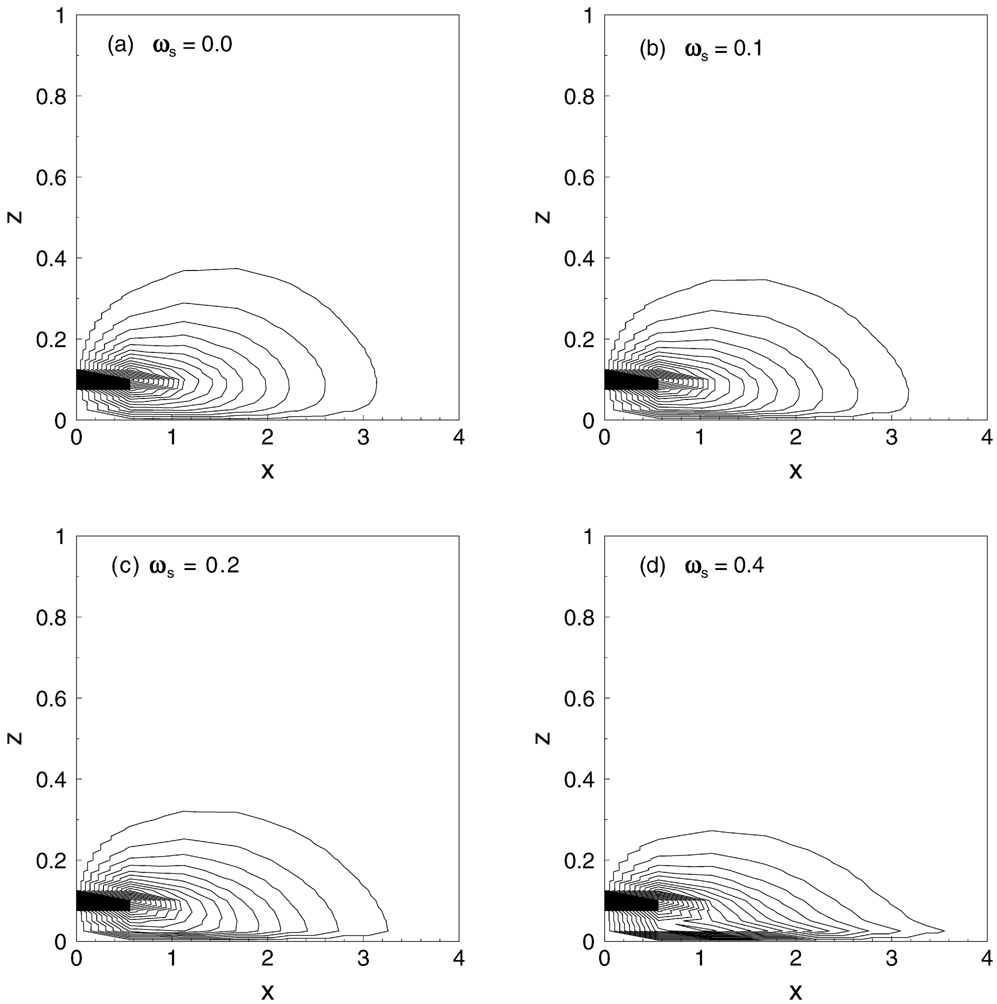

Fig. 8. Contours of equi-concentration in purely oscillatory flow of frequency $\Omega = 4$ in the xz-plane for unsteady dispersion of the particles released at a height $z_s = 0.11$ and for different settling velocities at time $t = 0.6$. The strength of the concentration of settling particles is 0.01 at the outermost contour.

This system of equations are solved for $C(j,k,n+1)$, $k = 1,\dots,M+1$, for each row $j = 2,\dots,N$. Also, an inverse transformation is used to get back to the physical plane from the computational plane.

4. Results and discussions

In order to validate the numerical scheme with the experimental data of Raupach and Legg [31] and the numerical results of Sullivan and Yip [32], a check has been made to the steady-state dispersion equation (11) subject to the boundary and prescribed input conditions with $\omega_s = 0$, considering the steady flow $u_0(z)$ given by Eq. (4) and the corresponding eddy diffusivity $k(z)$ given by Eq. (7). Raupach and Legg [31] performed their experiment on passive tracers by making a rough surface gluing 7 mm gravel to a wooden base board, where a heat source was situated at 60 mm above the zero plane of the surface and the depth D of the carrier fluid was 540 mm. Their measured data yielded an approximate logarithmic velocity profile $u = \frac{1}{\kappa}\ln(z/z_0)$ with roughness height $z_0^* = 0.12$ mm and $\kappa = 0.38$. The vertical and downstream distances were normalized by the heat source height h for the comparison

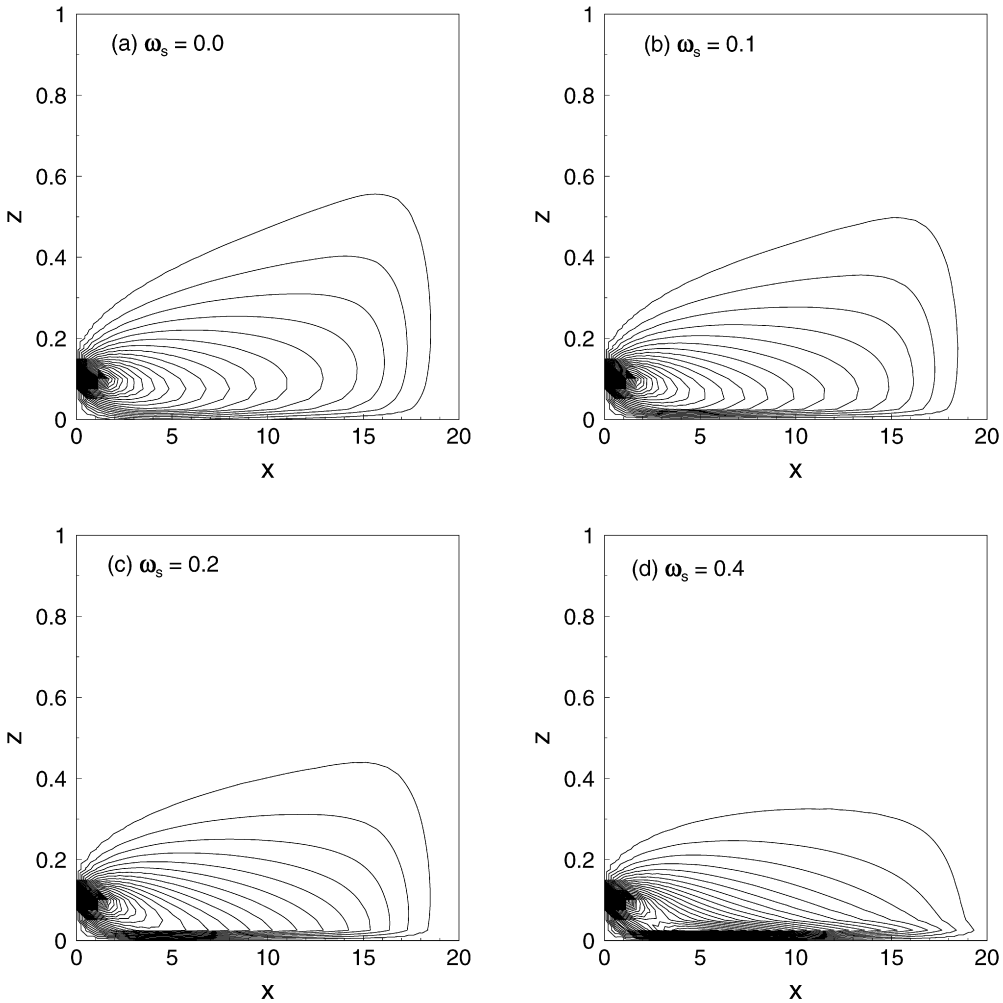

Fig. 9. Contours of equi-concentration in oscillatory flow with non-zero mean for unsteady dispersion. All other parameters are same as Fig. 8.

with the measured and computed concentration values. Following Raupach and legg [31], concentration C has been normalized by a temperature scale of the form:

$$\theta^* = \frac{F}{hu(h)}$$

where F is the constant flux of contaminant through a plane normal to the flow. Comparison of the present results with the experimental data of Raupach and Legg [31] and the numerical results of Sullivan and Yip [32] for the steady-state concentration equation at four downstream stations ($x^*/h = 2.5, 7.5, 15.0, 30.0$) are shown in Fig. 1(a)–(d). For better agreement of the present result with the experimental data of Raupach and Legg [31], the value of the wake-strength parameter $\Pi = 0.09$ is taken according to Song et al. [37]. Also, in this problem, we have taken $k_\zeta(\eta) = k_\eta(\eta)$ and stretching factor $a = 0.1$. A remarkably good comparison is achieved, except at the closest downstream station $x^*/h = 2.5$ (see Fig. 1(a)). This discrepancy may be due to a time-independent eddy-diffusivity approach to the closest downstream station or for neglecting the off-diagonal diffusion terms. However, a better representation of data only at the closest measuring station ($x^*/h = 2{\cdot}5$) from the source was achieved by Sullivan and Yip [38] incorporating a some sort of time dependent eddy-diffusivity. Therefore, the present numerical scheme may be extended for long-time diffusion of settling particles in an oscillatory flow with or without a non-zero mean using time independent eddy-diffusivity (Mei and Chian [25]). Normalized steady concentration profiles of suspended particles of different settling velocities (ω_s = 0.1, 0.2, 0.4) are shown in Fig. 2(a)–(d) for purely oscillatory flow of frequency $\Omega = 4$

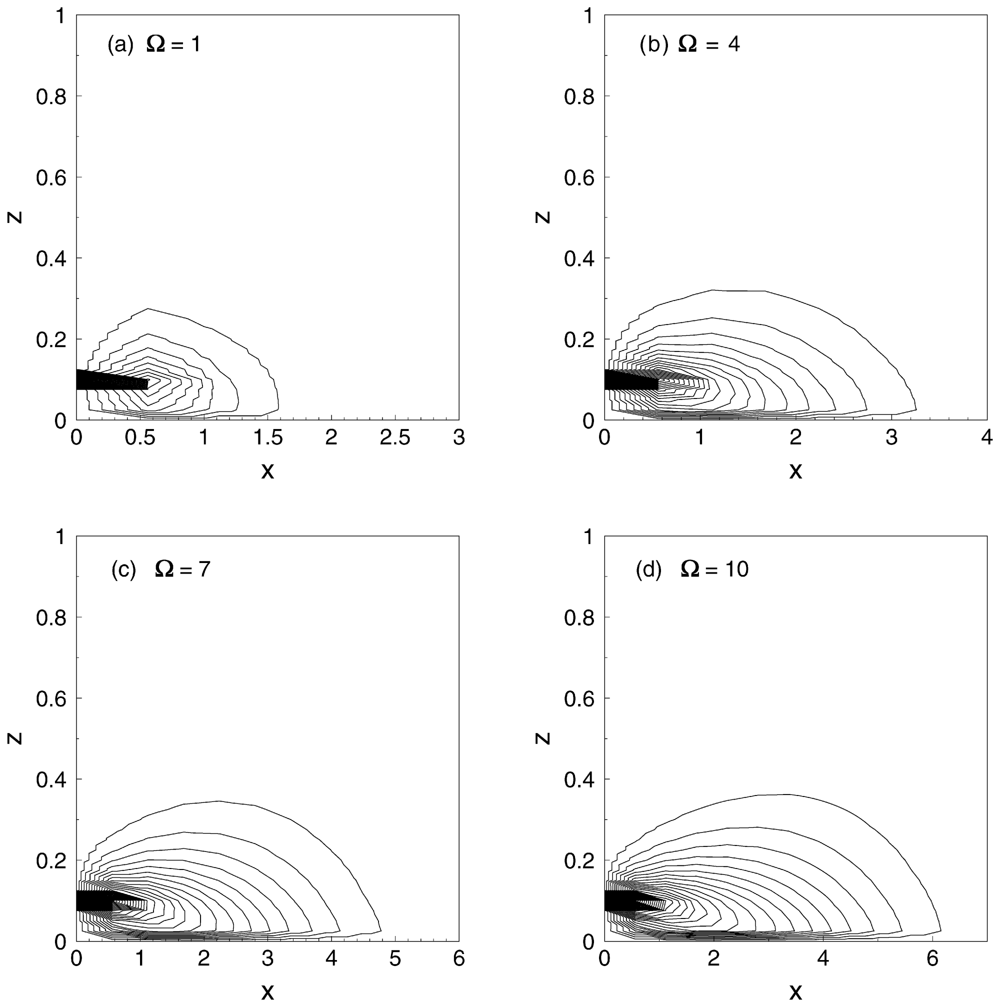

Fig. 10. Iso-concentration contours for unsteady dispersion of the particles of settling velocity $\omega_s = 0.2$ released at a height $z_s = 0.11$ in purely oscillatory flow at non-dimensional time $t = 0.6$ for different frequency parameters. The strength of the concentration of settling particles is 0.01 at the outermost contour.

and in Fig. 3(a)–(d) for oscillatory flow with non-zero mean in various downstream distances at time $t = 2.0$. It is observed that increase of settling velocity leads to increase of concentration near the bed surface. This may be explained by the fact that the particles with high settling velocities have a tendency to fall towards bed as it proceeds towards downstream. In the case of oscillatory flow with non-zero mean, the deposition of settling particles decreases significantly near the bed surface compared to that of the purely oscillatory flow. The reason is that the mean flow carries the suspended particles along the downstream and the mixing of the settling particles due to the mean flow along with perturbation was large compared with that of oscillatory flow. The dimensionless steady concentration profiles of settling particles ($\omega_s = 0.2$) at various downstream stations for different values of frequency parameter (Ω = 4, 7, 10) are depicted in Fig. 4(a)–(d) for purely oscillatory flow and in Fig. 5(a)–(d) for the oscillatory flow with non-zero mean at time $t = 2.0$. It is clearly observed from the figures that the elongation in the concentration profiles of the suspended particles decreases with increase of frequency. It may be explained by the fact that increase of frequency of tidal oscillation leads to decrease of concentration near the bed surface, which implies more dispersion of suspended particles. The similar behaviour of vertical concentration profiles is observed but the elongation of the profiles for oscillatory flow with non-zero mean is less compared with that of oscillatory flow.

Fig. 6(a)–(d) shows the time variation of iso-concentration contours for unsteady dispersion of the settling particles ($\omega_s = 0.2$) injected at a height $z_s = 0.11$ for purely oscillatory flow of frequency $\Omega = 4$. In each of the

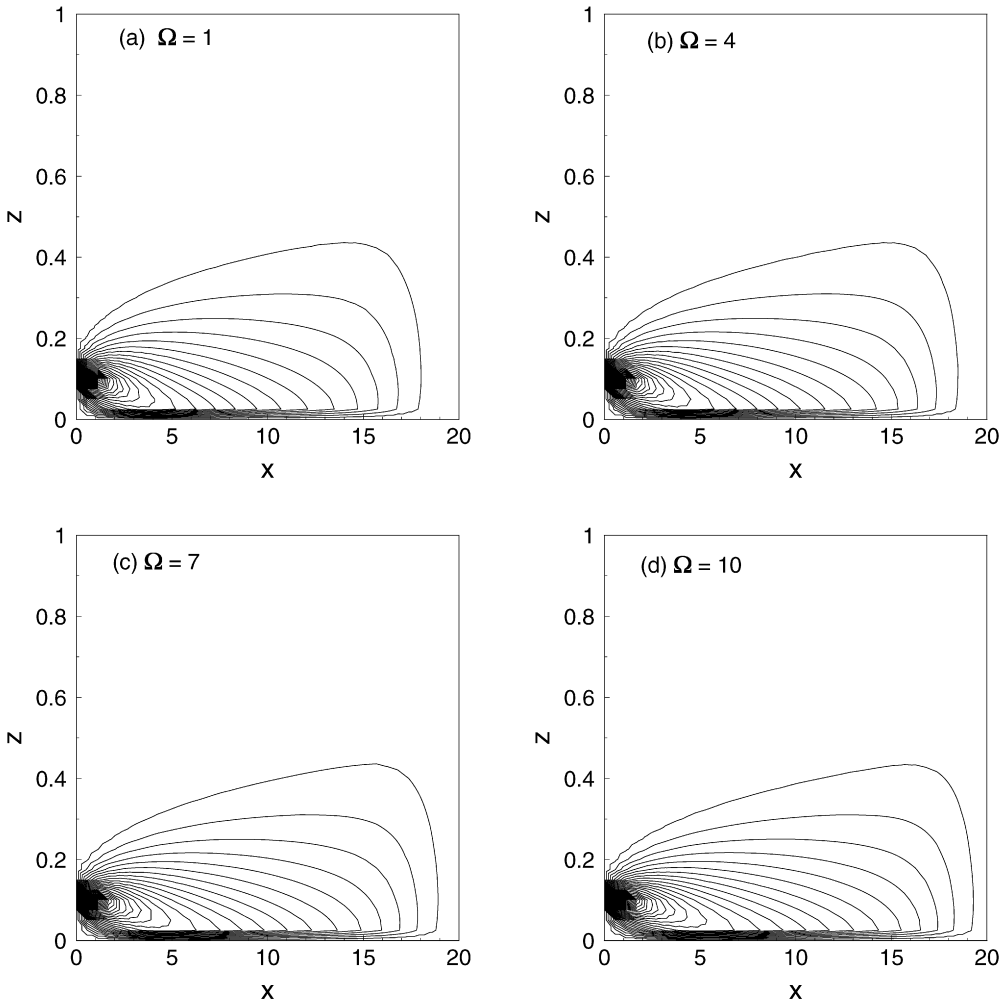

Fig. 11. Iso-concentration contours for unsteady dispersion of the particles in oscillatory with non-zero mean. All other parameters are same as Fig. 10.

iso-concentration contours the strength of the concentration of the settling particles at the outermost contour is 0.01. It is observed that the dispersion of suspended particles with settling velocity increases with time and subsequently, the concentration of settling particles increases at the bed surface. Also, it is clear that as time increases the particles settle to the bed. As a result the equi-concentration lines bend along the bed surface. In Fig. 7(a)–(d) the time variation of iso-concentration contours for unsteady dispersion of the settling particles is depicted for the same parameters as in Fig. 6(a)–(d) for the oscillatory flow with non-zero mean. From the figures it is clear that as the time increases the particles disperse significantly in the longitudinal as well as vertical directions and it is also observed that the concentration contours become more elongated in the longitudinal direction compared to vertical direction because dispersion due to longitudinal convection is much stronger than vertical eddy diffusivity (Sumer [18]). In Fig. 8(a)–(d), the contours of equi-concentration for unsteady dispersion are plotted for different values of settling velocity in purely oscillatory flow of frequency $\Omega = 4$ at time $t = 0.6$, when the material is released at a height $z_s = 0.11$. It is seen that the near-bed iso-concentration lines show agglomeration due to the combined action of the oscillatory motion of the flow, the turbulence and gravity when settling velocity ω_s increases. Fig. 9(a)–(d) shows the contours of equi-concentration for unsteady dispersion in oscillatory flow with non-zero mean for the same values of the parameters as in Fig. 8(a)–(d). It is seen that as the settling velocity increases, the near-bed iso-lines of concentration show a complex behaviour and the dispersion is much more than that for purely oscillatory flow.

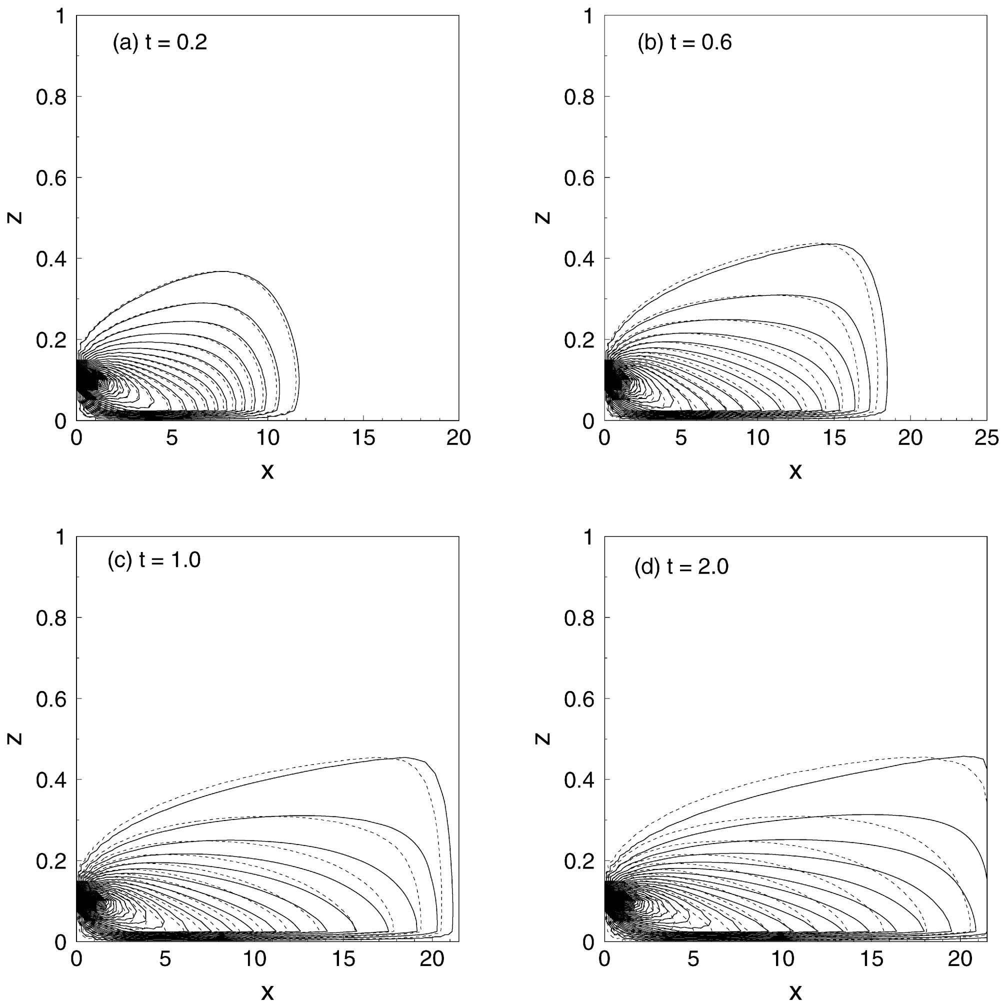

Fig. 12. Iso-concentration contours for unsteady dispersion of the particles of settling velocity $\omega_s = 0.2$ released at a height $z_s = 0.11$ in oscillatory flow of frequency $\Omega = 4$ with non-zero mean (solid lines) and steady flow (dotted lines) for different non-dimensional dispersion times. The strength of the concentration of settling particles is 0.01 at the outermost contour.

The iso-concentration contours for unsteady dispersion of released particles of settling velocity ($\omega_s = 0.2$) for different values of frequency parameter ($\Omega = 1, 4, 7, 10$) at time $t = 0.6$ are depicted in Fig. 10(a)–(d) for purely oscillatory flow and in Fig. 11(a)–(d) for the oscillatory flow with non-zero mean, when the particles are released at the height $z_s = 0.11$ above the bed surface. It is observed from Figs. 10 and 11 that as the tidal frequency increases the dispersion of the settling particle increases and subsequently the elongation of the concentration contour in the xz-plane increases. But the rate of increment for the case of oscillatory flow with non-zero mean (see Fig. 11(a)–(d)) is significantly higher than that of purely oscillatory flow. It is interesting to note that for all iso-concentration lines in case of purely oscillatory flow some sort of fluctuations near the source is observed due to tidal oscillation whereas for the case of non-zero mean, the effect of tidal frequency becomes insignificant, and consequently dispersion increases due to the superposition of perturbation to the mean flow. Fig. 12(a)–(d) shows the comparative study between the iso-concentration of settling particles ($\omega_s = 0.2$) for steady (Mondal and Mazumder [22]) and oscillatory flow of frequency ($\Omega = 4$) with non-zero mean at different times. It is observed that the elongation in the iso-concentration contours increases with time for both the cases. The superposition of perturbed flow to the mean flow leads the contour to be expanded little more than that of steady flow only, but the nature of the contours is almost same. The iso-lines of concentration of settling particles ($\omega_s = 0.2$) released at two different heights ($z_s = 0.31, 0.61$) for two dispersion times ($t = 0.6, 1.00$) have been plotted in Fig. 13(a)–(b) for oscillatory flow with frequency $\Omega = 4$ and

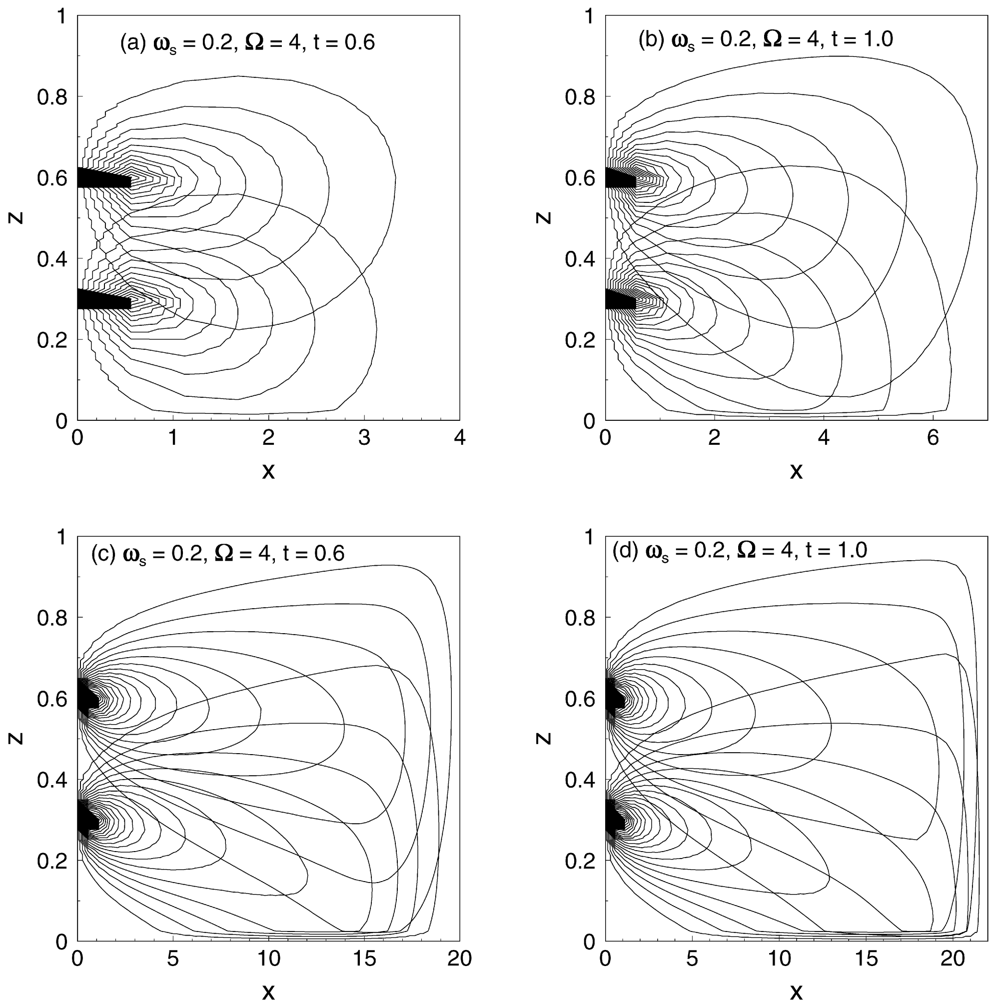

Fig. 13. Iso-lines of concentration for unsteady dispersion of the particles of settling velocity $\omega_s = 0.2$ released at heights $z_s = 0.31$ and 0.61: (a), (b) for purely oscillatory flow of frequency $\Omega = 4$ with times $t = 0.6$ and 1.0 respectively; (c), (d) for oscillatory flow of frequency $\Omega = 4$ with non-zero mean at times $t = 0.6$ and 1.0 respectively. The strength of the concentration of settling particles is 0.01 at the outermost contour.

in Fig. 13(c)–(d) for oscillatory flow with non-zero mean. It is seen that in both cases as dispersion time increases the elongation of the contours increases and it bends bottom due to the effect of the settling velocity. It is interesting to note that for purely oscillatory flow the variations of the contour lines with times $t = 0.6$ and $t = 1.0$ increases significantly but for oscillatory flow with non-zero mean it advances slightly. The fact is that, for purely oscillatory flow the time taken for reaching a stationary state of dispersion is much greater than that of oscillatory flow with non-zero mean (Bandyopadhyay and Mazumder [39]).

5. Conclusions

(1) The stream-wise dispersion of settling particles has been studied in an oscillatory turbulent flow when the particles are released from an elevated continuous line-source. Using a modified 'log-wake law' and the corresponding eddy diffusivity, the present numerical results are compared with the existing experimental and numerical results for steady-state of the convection–diffusion equation when settling velocity is zero and are found in good agreement except at the closest downstream station.
(2) It is observed that as setting velocity increases, the concentration of settling particles increases at the near bed surface for both oscillatory and oscillatory flow with a non-zero mean.

(3) Normalized iso-concentration profiles for oscillatory flow with a non-zero mean decreases significantly near the bed compared to that of oscillatory flow.
(4) For two-dimensional unsteady dispersion, the results are discussed in the form of iso-concentration contours in the vertical xz-plane in terms of the relative importance of convection, eddy diffusion and settling velocity. It is found that as settling velocity increases, the near-bed iso-concentration lines become agglomerated due to the interaction of the mean shearing motion of the fluid, turbulence and gravity for both oscillatory and the oscillatory with non-zero mean flows.
(5) Comparison of iso-concentration lines with time for steady and oscillatory with non-zero mean flows shows that the elongation in the iso-concentration contours increases with dispersion time and the contours for oscillatory flow with non-zero mean slightly advances due to the presence of perturbed part than that for the steady flow but the features of the contours are almost similar. Also, it is interesting to note that for purely oscillatory flow the time for reaching a stationary state of dispersion is much greater than that of the oscillatory flow with non-zero mean.

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